Determining Servo Torque Requirements
(or something I did to occupy myself during the pandemic lockdown)
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Have you ever had a concern whether the torque capability of the servos you choose for a given model is adequate? If we are building a model from a kit or an ARF, quite often there will be recommended servos to use with a consequent stated torque capability. If you do not want to use the recommended servos you can always choose something different and use the recommended servo as a guide for the torque required. But what if you are building something from scratch that does not have something similar to compare with to determine the torque required?

There are Servo Torque Calculators on the web at:
https://www.radiocontrolinfo.com/information/rc-calculators/rc-airplane-calculator/#Torque

and by Chuck Gadd, at: http://www.mnbigbirds.com/Servo%20Torque%20Caculator.htm

There is no means of verifying the first calculator above as it is form driven. Chuck Gadd's formula however, although it combines the drag coefficient, the air density, numerals and several unit conversion factors into one constant, does provide enough of a basis for verification. I spent my working life in an industry where one does not take anything for granted unless it is verified to the nth degree. So I set about developing my own formula for comparison.

If you are interested in the development of the formula, it is given in an Appendix to this issue of TASK. The formula is as follows:

$$Ts = Cd\rho V^2 LC^2 \sin\alpha h \tan\alpha h / 4\tan\alpha s$$

Where:
- ***Ts*** is the servo torque required in Newton-meters
- ***Cd*** is the drag coefficient of the control surface at its greatest deflection. I suggest take this as 1.0, except for a long aspect ratio flap at rotations greater than 60^0. The rationale for ***Cd*** values is given in the Appendix.
- ***ρ*** is the air density in kg/m^3 Take this is 1.2 kg/m^3. This is conservative for where SOGGI flies.
- ***V*** is the airspeed in meters/second
- ***L*** is the control surface length in meters
- ***C*** is the control surface chord in meters
- ***αh*** is the rotation angle of the control surface from neutral in degrees
- ***αs*** is the rotation angle of the servo arm in degrees measured from the servo arm position at 90^0 to the pushrod

Servo torque is usually specified in oz-in or kg-cm. To obtain the torque in oz-in or kg-cm, multiply the result in N-m by 141.6 or 10.2 respectively.

Ts (oz-in) = ***Ts*** (N-m) x 141.6
Ts (kg-cm) = ***Ts*** (N-m) x 10.2

If you use an air density of 1.2 kg/m^3 and a drag coefficient of 1.0, the formula above gives an identical result to Chuck Gadd's formula, and a result about 5% lower than the calculator on the first website above.

For anyone who wants to use it, I have a spreadsheet for the above formula. The spreadsheet is very useful as it enables one to quickly vary parameters for different airspeeds, control surface geometries, servo and surface rotations, to obtain servo torque required. The spreadsheet also enables unit conversions for units commonly used for control surface dimensions and airspeed.

Typically the servo torque recommended by model manufacturers is higher than would be determined using the above formula, which is expected. The formula does not allow for friction between the pushrod and mating parts, or at the hinge. Although any friction is typically very small, the servo torque available should nevertheless allow for some margin above that calculated.

Here are some tips for optimizing servo torque and control surface movement.

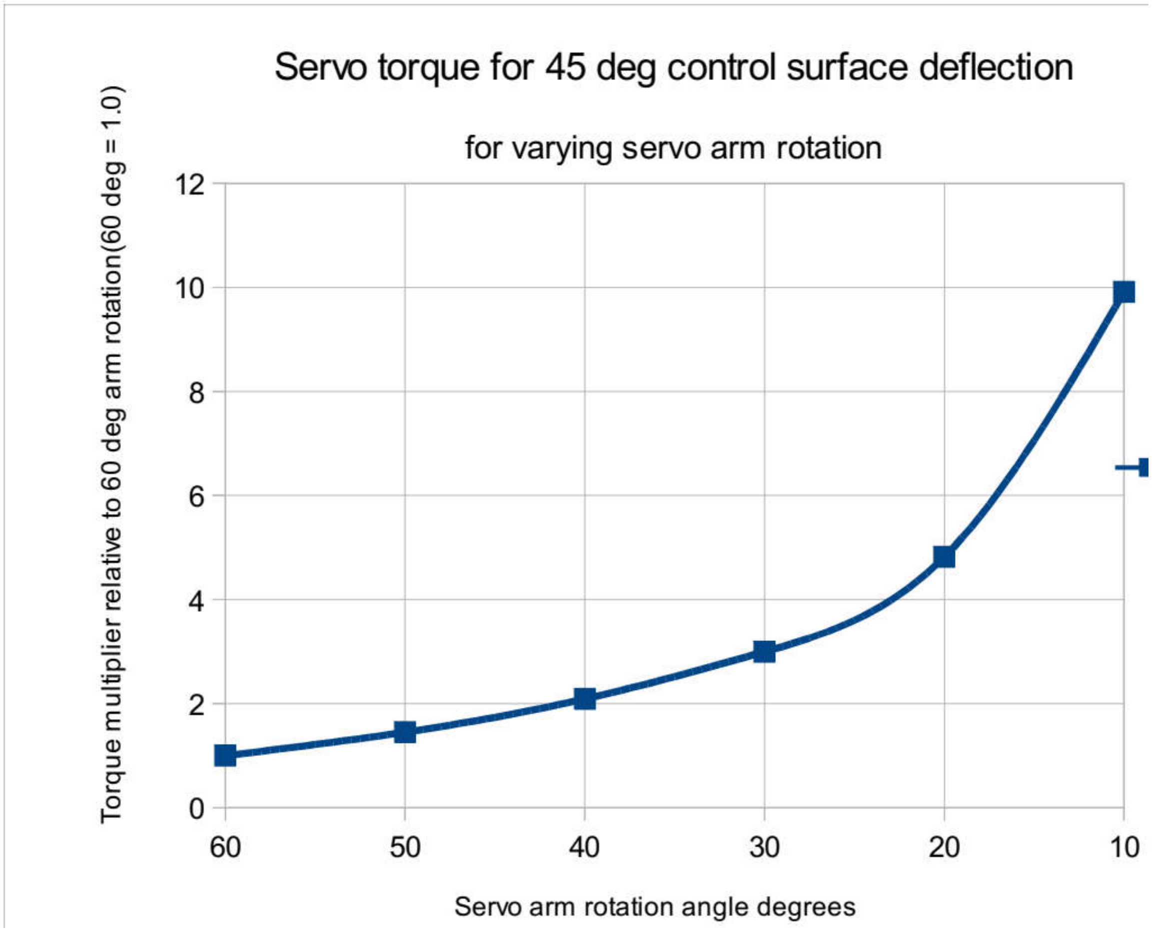
- For control surfaces that move approximately equal distances either side of neutral, the servo arm should be at 90^0 to the pushrod at the neutral position. And the control surface horn hole should be as close as possible to 90^0 to the control surface at the hinge line.

- For control surfaces that move predominately in one direction, e.g. flaps, optimize the servo arm position so that it is at 90^0 to the pushrod at half of the control surface movement. This maximizes the rotation of the servo arm and minimizes the torque required.

- If you need to reduce the torque required for a given control surface movement, shorten the servo arm relative to the control horn, i.e. servo arm rotation angle is high relative to horn rotation angle. Although this is at a penalty of reduced control surface movement for a given servo arm rotation, it will save battery power. Set up your servo to control surface linkage arrangement to maximize servo arm movement if this still enables you to achieve the desired control surface movement.

- If you need a larger control surface movement lengthen the servo arm relative to the control horn, i.e. servo arm rotation angle is low relative to horn rotation angle. But this is at a penalty of higher servo torque required.

The bottom line is; for a given control surface movement, the larger the servo arm rotation angle (i.e. the shorter the servo arm relative to the horn length) the lower the servo torque required. This is illustrated in the graph below. This assumes a 45^0 control surface deflection and shows the increase in servo torque required for reducing servo arm rotation relative to a 60^0 arm rotation. If the servo arm rotation is decreased from 60^0 to 40^0, the torque required doubles. If the arm rotation is decreased to 10^0, the torque required is nearly 10 times higher.



Send Andy an email if you want a copy of the servo torque calculator spreadsheet.

<u>Appendix: Servo Torque Formula Development</u>

Nomenclature	Parameter, see Figure 1	Unit
Fa	aerodynamic drag force on control surface	N (Newtons)
Cd	drag coefficient of a rectangular plate	Dimensionless
$\boldsymbol{\rho}$	air density	kg/m3
V	airflow velocity	m/s
A	projected area of the control surface perpendicular to the airflow	m^2
Th	torque on the control surface and horn	Nm
Ts	torque on the servo arm	Nm
$\boldsymbol{\alpha h}$	control surface/horn rotation angle	degrees
$\boldsymbol{\alpha s}$	servo arm rotation angle for above control surface horn rotation angle	degrees
Fp	force in line with the control pushrod	N
L	control surface length	m
C	control surface chord	m
P	pushrod axial movement for above control surface and servo arm rotations	m
xh	effective moment arm of control surface horn	m
xs	effective moment arm of servo arm	m

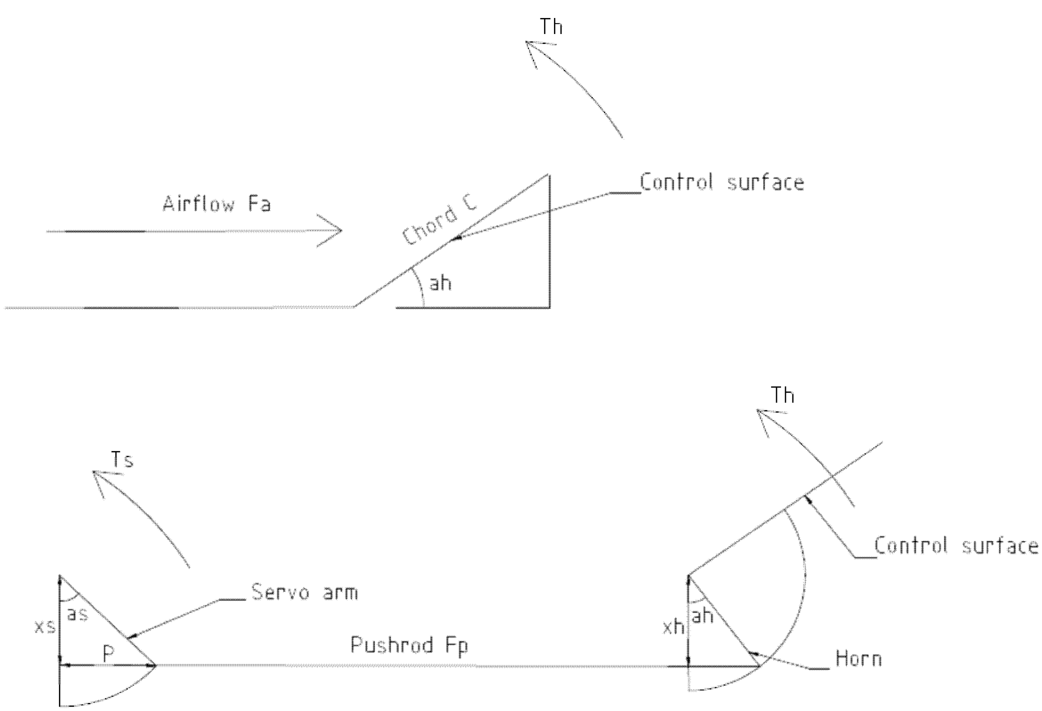


Figure 1

The aerodynamic drag force on a body is given by the following formula:

$$Fa = (Cd\rho V^2 A)/2 \qquad (1)$$

The projected area of the control surface presented to the airflow is zero when the control surface is at the neutral position, and the full area (i.e. LxC) when the surface is at 90^0 to the airflow. We are typically only interested in when the control surface is at intermediate positions, for which the area normal to the airflow is:

$$A = LC\sin\alpha h \qquad (2)$$

Csinαh being the raised height of the control surface at 90^0 to the airflow.

The torque on the control surface, which is the same as that on the control horn is:

$$Th = FaC/2^* \qquad (3)$$

C/2 being the average chord over which the drag force Fa, evenly distributed over the control surface acts. *see footnote

The torque on both the control surface horn and servo arm vary with the respective rotation angles of the horn and servo arm, because the moment arms vary with the rotation angles. This is not as simple as two gearwheels connected where the moment arms for each gearwheel are constant.

The torque on the control surface horn and the torque on the servo arm are related via the force Fp along the connecting pushrod.

The torque on the control surface horn is:

$Th = Fpxh$ and

$xh = P/\tan\alpha h$ hence substituting for xh in the above

$Th = Fp\ P/\tan\alpha h$ so

$$Fp\ P = Th\ \tan\alpha h \qquad (4)$$

Assuming the pushrod is long relative to the horn and servo arm, the movement of the pushrod P is the same for the arm and horn. So similarly the torque on the servo arm is:

$Ts = Fpxs$ and

$xs = P/\tan\alpha s$ hence substituting for ***xs*** in the above

$$Ts = Fp\ P/\tan\alpha s \qquad (5)$$

Substituting (4) in (5) gives:

$$Ts = Th \tan\alpha h / \tan\alpha s \quad (6)$$

Hence the relationship between the torque on the control surface/horn and that of the servo is the ratio of the tangents of their respective angular rotations.

What we are interested in of course is the servo torque as a function of the other parameters that we know, can measure or estimate.

So substituting (3) in (6) for ***Th*** gives:

$$Ts = FaC \tan\alpha h / 2\tan\alpha s \quad (7)$$

Substituting (1) in (7) for ***Fa*** gives:

$$Ts = Cd\rho V^2 AC \tan\alpha h / 4\tan\alpha s \quad (8)$$

Substituting (2) in (8) for the control surface area ***A*** gives:

$$\boxed{Ts = Cd\rho V^2 LC^2 \sin\alpha h \tan\alpha h / 4\tan\alpha s} \quad (9)$$

We know ρ, can estimate ***Cd*** and ***V***, and measure ***L***, ***C*** and the rotation angles.

The drag coefficient is complicated and dependant upon; the control surface aspect ratio, the inclination angle (i.e. control surface rotation) and the Reynolds number (Re). The drag coefficient of an inclined rectangular plane of short aspect ratio is 1.28 times the sine of the inclination angle. So using this relationship, ***Cd*** would be 0.22 at 10^0 inclination, 0.98 at 50^0 and 1.28 at 90^0. The ***Cd*** of a large aspect ratio rectangle at 90^0 is 2.0, but I could not find any data giving ***Cd*** between these extremes. I would choose a ***Cd*** of 1.0 which conservatively covers most control surface geometries up to 50^0 of surface inclination. However, in the case of a long aspect ratio flap deflected close to 90^0 to the airflow it would be appropriate to use a larger value. A ***Cd*** of up to 2.0 would be conservative in this case.

We can take the air density as 1.2 kg/m^3 which is slightly higher than that (1.16 kg/m^3) at the ground level we fly at in the Hamilton area and at a typical flying season temperature of 25^0 C. As one flies higher, the density reduces, but is likely compensated by higher airspeeds at the lower density.

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* I am open to the argument that $Th = FaC\sin\alpha h/2$ should be the horn torque, $C\sin\alpha h/2$ being the moment arm perpendicular to the drag force in the direction of the airflow. But this does not agree with Gadd's formula and results in very low servo torque values. Using $C/2$ as opposed to $C\sin\alpha h/2$ is conservative.